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Marie-Laure CABON-DHERSIN, Emmanuelle TAUGOURDEAU

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Research clusters: How public subsidies matter?*

Marie-Laure Cabon-Dhersin†, Emmanuelle Taugourdeau‡

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Abstract

This paper investigates the factors underlying the emergence of Research Cluster (RC), i.e. cooperation (or coordination of research efforts) through spatial proximity between public and private research teams. A 'public lab' and a 'private lab' interact in a two-stage game to decide on 'location' and 'research effort'. A high level of public subsidies associated to a low asymmetry in the 'valorisation capability' between both labs is necessary for the formation of a cluster. We find that RC performs better than non-cooperation in terms of research efforts in a 'public lab' (but not in a 'private lab') and output gains that can be appropriated by each lab.

Key words: research cooperation, spatial location, public subsidy.
Code JEL: C7, H2, H4, L3, L5, 03.

1 Introduction

A recurrent challenge faced by European authorities is how to establish closer cooperation between universities and firms to ensure greater production and

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†Corresponding author: University of Rouen, CREAM, 3 avenue Pasteur, 76100 Rouen, France. (marie-laure.cabon-dhersin@univ-rouen.fr)

‡CNRS, CES, Ens Cachan, 61 avenue du President Wilson, 94230 Cachan, France. (emmanuelle.taugourdeau@ens-cachan.fr)

better dissemination of new knowledge (European-Commission, 2003, 2008). Regional knowledge clusters, competitiveness clusters, poles of excellence and/or Science and Technology parks reflect such new institutional arrangements that facilitate interactions between universities and industries (Link and Scott, 2007).

These initiatives to nurture competitiveness stem from theoretical approaches and empirical investigations on the advantages of R&D cooperation in the presence of knowledge spillovers (i.e. knowledge produced by one agent that may contribute to the production of knowledge by the other agents without cost). Theoretical models on R&D cooperation show that coordination of research efforts between competitive firms may help firms to internalize their knowledge spillovers, resulting in greater research efforts and higher social welfare (d'Aspremont and Jacquemin, 1988; Kamien et al., 1992; De Bondt, 1997; Amir, 2000; Norman and Pepall, 2004). This argument can be applied to research partnerships between firms and universities as well (Poyago-Theotoky, 2009; Poyago-Theotoky et al., 2002; Beath et al., 2003). Since knowledge has the properties of a public good, outcomes of public sector research has often been put to good use by private sector firms (Jaffe, 1989; Audretsch et al., 2002; Cohen et al., 2002; Autant-Bernard, 2001; Cohen et al., 1994; Veugelers and Cassiman, 2005). For instance, firms using emerging technologies (e.g. bio- and nano-technology) have a strong interest in the knowledge produced by the public sector (Boufaden et al., 2007).

Others studies in the field of the geography of innovation have added an important spatial dimension to the discussion by illustrating that the positive effects also increase with proximity (Cooke, 2001; Furman et al., 2006; Autant-Bernard and LeSage, 2011). Geographical proximity which benefits the innovation process through diffusion of spillovers is frequently cited as an explanation for the emergence of cooperation between universities and firms (Audretsch, 1998; Varga, 2000; Audretsch et al., 2005).

What happens if we turn the question the other way around? Do firms and universities tend to move closer just in order to benefit from knowledge spillovers? This reverse causality is what we aim to examine in this paper. We also explore if cooperation performs better than non-cooperation in terms of research efforts and output gains that can be appropriated by research actors. When the private sector research does learn from public sector research through knowledge spillovers, which depend on the geographical proximity, it has an incentive to anchor cooperation at the same location in order to internalise the public externalities. Many cluster initiatives refer to Marshallian-type externalities. However, there is another force that works in the opposite direction. Since public and private research activities are rivals in the knowledge production and innovation, geographical proximity can also enhance competition between researchers of each sector. Recently, Bloom et al. (2013) and König et al. (2014) have shown that this competition effect (or rivalry effect) has a negative and significant impact on the payoff of the firms. This may also refer to a negative congestion externality, also known as "stepping on toes" effect. The race between public and private research in genome sequencing serves as an illustration of such competition (Carraro and Siniscalco, 2003).

Another phenomenon that we want to highlight in this paper is the impact of the geographical proximity on the size of the labs. Here, the 'size' of the lab emerges as an endogenous structure in terms of the number of researchers. However, the present model doesn't study the monetary incentives of labs to convince researchers to work for them. The stability of each team is given by the conditions of the standard coalition formation theory (d'Aspremont et al., 1983; Carraro and Siniscalco, 2003).

Let us refer to a 'Research Cluster' (or RC) as a set of public and private research organizations within close proximity to one another which practice cooperation. On positive grounds, this paper studies the existence conditions of such RC through a game theoretic formulation. Two actors 'public lab' and 'private lab' interact in a two-stage game to decide on 'location' and 'research effort'. We examine two scenarios: (i) in the first scenario, individual research efforts are chosen to maximize each lab payoff non cooperatively, (ii) in the second scenario, research efforts are chosen to maximize the joint payoffs. We examine whether cooperation facilitates spatial proximity and if it can yield a better performance than non-cooperation in terms of research efforts and gains that can be captured by each lab. We show that the results crucially depend on the level of the subsidy granted to the public lab together with the valorisation of the research efforts.

To deal with the incentive problem related to the production of public goods with externalities (spillovers), public lab is funded by a fixed subsidy provided by the government for each researcher, unrelated to research effort. Public lab can obtain a gain from commercialization and recognition related to research effort through patents, prizes, public recognition of the value of research and publications. Private lab can appropriate the research efforts by patent or copyright. We assume a one way movement of spillovers from the public lab to the private lab. The payoff of private lab depends on the collective efforts of its researchers as well as the exploitation of spillovers generated by public labs research efforts. In turn, closer proximity with public lab facilitates greater exploitation of spillovers. In our model, we assume realistically that for private lab the marginal benefit from its research effort is higher (or equal) than from research effort in public lab[1]. For this, we introduce the notion of 'valorisation capability' as the capability of a lab to appropriate monetary value from a research output. Solving for the Nash equilibrium, we show that 'location' and 'research effort' of each lab depend on the magnitude of the subsidy and the asymmetries in the 'valorisation capability' between public and private lab.

Our approach incorporates some ideas previously developed in economic geography (Fujita and Thisse, 2002). Some theoretical models examine how spillovers between firms shape the geography of production and innovation. They combine two different strands of literature: the theory of locational choice and the economics of innovation, dealing with spatial competition *à la* Hotelling (Biscaia and Mota, 2012; Belleflamme et al., 2000; Gersbach and Schmutzler, 1999) or *à la* Cournot (Van Long and Soubeyran, 1998). Mai and Peng (1999) introduce the element of cooperation between firms in the form of innovation exchange through communication into the Hotelling spatial competition model. Under the assumption that knowledge spillovers depend on firms' location, Piga and Poyago-Theotoky (2005) show that the distance between firms' location increases with the degree of product differentiation. In the same way, Van Long and Soubeyran (1998) obtain the result that firms agglomerate when the endogenous spillovers is a convex (or linear) function of firms' distance. In our model, we retain the assumption of a linear relationship between spillovers and distance, but only private lab chooses its location.

The contribution of the present work may be understood as follows. To the best of our knowledge, no existing spatial theoretical model has been proposed to explain the rationale for cooperative strategies at the same location between public and private research organizations[2]. On positive grounds, this paper is an attempt to fill this gap.

First, we explore the conditions on subsidy and "valorisation capability" levels allowing for the endogenous formation of a RC. For a given asymmetry in valorisation capability, the level of subsidy must lie between a minimum and a maximum level for a RC to be formed. A low level of the public subsidy is harmful for the co-existence of public and private research organizations: only the private research lab can exist whatever the level of asymmetry in valorisation capability. A high level of the public subsidy does not promote a close cooperation between the public and the private research labs: the size of the public lab decreases with the rise of the public subsidy reducing the

[1] In the public lab, knowledge resulting from research effort is traited as a partial public good and cannot be fully "owned".

[2] An exception is the paper by Mukherjee and Ramani (2011). The rationale for technology parks is explored between asymmetric firms.

positive spillover effect from the public lab. The private lab chooses to move away to avoid the negative competition effect. For the intermediate levels of subsidy, the private lab benefits from spillovers to come closer to the public lab and cooperative strategies facilitate spatial proximity.

Second, when comparing the payoff levels, we surprisingly find that the gain from the cooperative equilibrium dominates the gain from the non-cooperative equilibrium for a few couples of subsidy and "valorisation capability". Moreover, when focusing on the 'pure' RC (i.e. cooperation at the same location), we show that the 'pure' RC performs better than the non-cooperative equilibrium. However, this performance may be moderated by the results in terms of research efforts. Cooperation between the public and the private lab at the same location allows to encourage the public research effort but decreases research efforts in the 'private lab'.

The paper is organized as follows. Section 2 presents the model. Next in Section 3, we develop the results under two scenarios - competitive research and cooperative research-. We compare and discuss the results in Section 4. The final section concludes.

2 The model

Suppose that a research activity may be undertaken in two independent labs: a private sector lab (pr) and a public sector lab (pu). There exists a finite number N of researchers employed either by the public lab (n), or the private lab ($N-n$) with $0 \leq n \leq N$.

The game is modelled through two stages :

1. In the first stage, the private sector lab fixes the optimal distance from the public sector research lab on the segment $d \in [0, 1]$,

2. In the second stage, individual research efforts are chosen in each lab: x_i^{pu}, x_i^{pr}.

The size of each lab (number of researchers) results from the stability property of the equilibrium.

The research undertaken by the public lab may benefit the private lab at no cost (i.e. spillover effect) depending on location. The payoff of private lab depends on the collective efforts of its researchers as well as the exploitation of spillovers generated by the public lab research efforts. In turn, closer proximity with public lab facilitates greater exploitation of spillovers. We assume a one way movement of spillovers[3]. In the private research sector, the existence of intellectual property rights avoids knowledge spillovers in order to create a private incentive mechanism.

In the same line of Piga and Poyago-Theotoky (2005), we consider that the distance between both labs determines the size of the spillovers: spillovers are null, $1-d=0$, when the private lab locates at the location farthest from public lab, $d=1$; spillovers are maximum, $1-d=1$, when the private lab chooses to locate at the same point as the public lab, $d=0$.

The total payoff of the private research lab is given by:

$$\begin{cases} G^{pr}(x_i^{pr}) = w\sum_{1}^{N} x_i^{pr} - \sum_{1}^{N} (x_i^{pr})^2 \equiv G_A^{pr} \quad \text{if} \quad n=0 & (1) \\ G^{pr}(x_i^{pr}, x_i^{pu}) = w\sum_{n+1}^{N} x_i^{pr} + w(1-d)\sum_{1}^{n} x_i^{pu} \\ \qquad\qquad - \sum_{n+1}^{N} \frac{(x_i^{pr})^2}{(1+d)} \quad \text{if} \quad 1 \leq n \leq N-1 & (2) \end{cases}$$

where w is the marginal benefit yielded by the effective research effort in the private lab. The cost of research supported by each researcher depends quadratically on its research effort (x_i) and, as far as $n \neq 0$, negatively on the distance (d). The term $\frac{1}{1+d}$ captures the negative congestion externality, also known as "stepping on toes" effect or "product rivalry" effect, that may arise when researchers run parallel research programs[4]. Here, we consider that both labs being closer implies additional costs in terms of efforts to avoid competition effects. We can also interpret the impact of distance in the cost of effort as an efficiency measure of the effort, efficiency being lower in case of proximity between labs because of competition effects.

The public lab does not benefit from knowledge spillovers and the public lab payoff only depends on the sum of individual researchers efforts in pu. To deal with the incentive problem related to the production of public goods with externalities (spillovers), public lab is funded by a fixed subsidy, denoted s, provided by the government for each researcher, unrelated to research effort. The total payoff from the public research lab is given by:

[3]This assumption is based on empirical studies (Jaffe, 1989; Audretsch et al., 2002; Cohen et al., 2002; Autant-Bernard, 2001; Cohen et al., 1994; Veugelers and Cassiman, 2005)

[4]These effects have been clearly highlighted in empirical studies such as Bloom et al. (2013) and König et al. (2014).

$$\begin{cases} G^{pu}(x_i^{pu}) = ns + \dfrac{w}{a}\displaystyle\sum_1^N x_i^{pu} - \sum_1^N (x_i^{pu})^2 \equiv G_A^{pu} \quad \text{if} \quad n = N & (3) \\ G^{pu}(x_i^{pu}, x_i^{pr}) = ns + \dfrac{w}{a}\displaystyle\sum_1^n x_i^{pu} - \sum_1^n \dfrac{(x_i^{pu})^2}{1+d} \quad \text{if} \quad 1 \leq n \leq N-1 & (4) \end{cases}$$

where $\frac{w}{a}$ is the marginal benefit yielded by the effort of each researcher working in public lab. It represents the gain from commercialization and recognition related to research effort through patents, prizes, public recognition of the value of research, publications.

Here, we introduce the notion of 'valorisation capability' as the capability of a lab to appropriate monetary value from a research output. a is a parameter which illustrates the asymmetries in the 'valorisation capability' of research-effort outcomes for each lab. We assume realistically that for private lab the marginal benefit from its research effort is equal or higher than from research effort in public lab: $a \geq 1$. When a is equal to 1, the public lab and the private lab develop the same policy of adding value to their research effort.

3 The outcome at equilibrium

In this section we determine the non-cooperative and cooperative outcomes that we will compare in the next section. The two stages are non-cooperative. At the second stage, labs know the choices made at the preceding one.

3.1 Non-cooperative research between public and private sectors labs

Let us start with the analysis of the non-cooperative scenario (denoted by NC). The game is solved backward. The optimal level of effort is obtained by maximizing the payoff functions (2) and (4) with respect to x_i^{pr} and x_i^{pu} respectively taking the distance between labs and the number of researchers in each lab as given. This yields

$$\frac{\partial G^{pr}}{\partial x_i^{pr}} = 0 => x_{i,nc}^{pr} = w\frac{(d+1)}{2} \quad (5)$$

$$\frac{\partial G^{pu}}{\partial x_i^{pu}} = 0 => x_{i,nc}^{pu} = \frac{w}{a}\frac{(d+1)}{2} \quad (6)$$

We deduce collective research efforts in each lab:

$$X^{pr}_{nc} = (N - n_{nc})x^{pr}_{nc} = (N - n_{nc})w\frac{(d+1)}{2} \tag{7}$$

$$X^{pu}_{nc} = n_{nc}x^{pu}_{nc} = n_{nc}\frac{w}{a}\frac{(d+1)}{2} \tag{8}$$

Individual efforts depend positively on distance between labs due to the competition effect. Whatever the distance between labs, it is obvious that at the non cooperative equilibrium the individual effort in the public lab is lower than the individual effort in the private one:

$$x^{pu}_{nc} < x^{pr}_{nc}$$

Replacing these individual effort values into Equation (2) gives the expression of the private lab payoff function in the second stage of the game:

$$G^{pr}_{nc} = (w)^2\,\frac{(d+1)}{2}\left(\frac{1}{2}(N-n) + \frac{n}{a}(1-d)\right)$$

The private lab chooses the optimal distance, d_{nc}, from the public lab by maximizing its payoff function with respect to the distance d.

$$\frac{\partial G^{pr}_{nc}}{\partial d} = 0 \Leftrightarrow \underbrace{\underbrace{N-n}_{\text{effort effect}} + \underbrace{n(\frac{1-d_{nc}}{a} - \frac{d_{nc}+1}{a})}_{\text{spillover effect}}}_{\text{net marginal benefit}} = \underbrace{\underbrace{(N-n)}_{\text{effort effect}} - \underbrace{\frac{1}{2}(N-n)}_{\text{competition effect}}}_{\text{marginal cost}} \tag{9}$$

$$\Longleftrightarrow \underbrace{\frac{1}{2}(\frac{N}{n}-1)}_{\text{competition effect}} = \underbrace{\frac{1}{a}(2d_{nc})}_{\text{spillover effect}} \tag{10}$$

This maximization gives the following lemma:

Lemma 1. *For a given n, the optimal distance d_{nc} verifies the following properties:*

- $d_{nc} = \frac{a(N-n)}{4n}$ *where* d_{nc} *is a solution of* $\frac{\partial G^{pr}_{nc}}{\partial d} = 0$ *if and only if* $n \geq N\frac{a}{4+a}$ *with* $\frac{\partial d_{nc}}{\partial n} = -\frac{aN}{4n}$
- $d_{nc} = 1$ *otherwise*

- *$\forall n$, the solution is unique :* $\frac{\partial^2 G^{pr}_{nc}}{\partial d^2} = -\frac{2n}{a} < 0$

Proof: in Appendix 1. □

For a given number of researchers in each team, the distance between both labs is the one that balances the marginal cost with the net marginal benefit due to the marginal increase in distance as Equation (9) shows. The right hand side of Expression (9) highlights the cost through two effects: when the distance between both teams rises, it increases the cost due to the research effort but it diminishes the cost due to the competition effect. In the left hand side of Expression (9), two other opposite effects can be observed: (i) indirectly, the distance increases the individual research efforts (effort effect), (ii) the last effect characterizes the combination of spillover effects. The increase of the distance between both teams diminishes the spillovers towards the private team $\left(-\frac{n}{a}(d+1)\right)$ but increases also the effort of the public researchers and so the spillovers from the public lab $\left(\frac{n}{a}(1-d)\right)$. The total spillover effect is negative. At the optimum, the competition effect which only depends on the number of researchers n is equal to the loss from an increase of distance (spillover effect) (Equation 10).

The distance depends negatively on the number of researchers in the public lab because when this number increases, it diminishes the effort effect by more than twice the cost of competition effect whereas it increases the spillover effect which is negative. The effect of n on the marginal private payoff is clearly negative. In order to restablish the balance between the marginal net benefit from effort and the marginal loss from spillover effect, the private lab must choose to locate closer to the public team (diminution of d), which reduces the negative spillover effect.

Following the standard coalition formation theory (d'Aspremont et al., 1983; Carraro and Siniscalco, 2003), we define the equilibrium as follows:

Definition 1. *The two-lab Nash equilibrium is defined by the following conditions:*

$$G^{pu}(n^*) > 0 \quad \text{and} \quad G^{pr}(n^*) > 0, \tag{11}$$
$$G^{pr}(n^*) \geq G^{pu}(n^*-1) \quad \text{and} \quad G^{pu}(n^*) \geq G^{pr}(n^*+1) \tag{12}$$

where $0 < n^* < N$.

At this equilibrium, the private lab does not benefit from an additional researcher and the public lab does not benefit that a researcher leaves its team to join the private lab.
Assuming N sufficiently large, conditions (11) and (12) can be approximated by the following non-cooperative equilibrium condition:

$$G_{nc}^{pu}(n_{nc}^*) = G_{nc}^{pr}(n_{nc}^*) \tag{13}$$

In order to verify the stability property of the equilibrium, we have to study both interior and corner solutions for d_{nc}.

For the interior solution, we rewrite the payoff functions according to the previous stages of the game:

$$G_{nc}^{pu} = n_{nc}\left[s + \frac{1}{4}\left(\frac{w}{a}\right)^2 (1 + d_{nc})\right] \tag{14}$$

$$G_{nc}^{pr} = \frac{n_{nc}}{2}\frac{(w)^2}{a}(1 + d_{nc})^2 \tag{15}$$

For the corner solution, the payoff functions rewrite

$$G_{nc}^{pu}(d_{nc} = 1) = n(1)\left[s + \frac{1}{2}\left(\frac{w}{a}\right)^2\right] \tag{16}$$

$$G_{nc}^{pr}(d_{nc} = 1) = \frac{N - n(1)}{2}(w)^2 \tag{17}$$

Determining the equilibrium of the game, we obtain the following proposition

Proposition 1. *Under NC scenario, the subgame perfect Nash equilibrium is such that:*

1. *If the public subsidy s is "too low" ($s < \underline{s}$), only private research lab exists, $n^* = 0$.*

2. *If the public subvention s is equal to $\underline{s}$, only public research lab exists, $n^* = N$.*

3. *If the public subsidy s is neither "too high" ($s \leq \bar{s}$) nor "too low" ($s > \underline{s}$), both research labs coexist with:*

$$n_{nc}^* = N\frac{a}{4d_{nc}^* + a} \quad \textit{with} \quad N\frac{a}{4 + a} \leq n_{nc}^* < N$$

and

$$d_{nc}^{*}=\frac{1}{4a}\left[1-4a+\sqrt{1+32sa\left(\frac{a}{w}\right)^{2}}\right]\in(0,1]$$

4. *If the public subsidy s is "too high" ($s>\overline{s}$), both research labs coexist with:*

$$n^{*}(1)=N\frac{\frac{1}{2}(w)^{2}}{s+\frac{1}{2}\left(\frac{w}{a}\right)^{2}(1+a^{2})}\quad \textit{with}\quad 0<n^{*}(1)<N\frac{a}{4+a}$$

and

$$d_{nc}^{*}=1$$

With

$$\underline{s}=\left(\frac{w}{a}\right)^{2}\frac{2a-1}{4}\ \textit{and}\ \overline{s}=\left(\frac{w}{a}\right)^{2}\frac{4a-1}{2}.$$

Proof: in Appendix 2. □

Corollary 1. *Under NC scenario, the "pure" RC ($d_{nc}^{*}=0$ with both labs) may not emerge as an equilibrium of the game.*

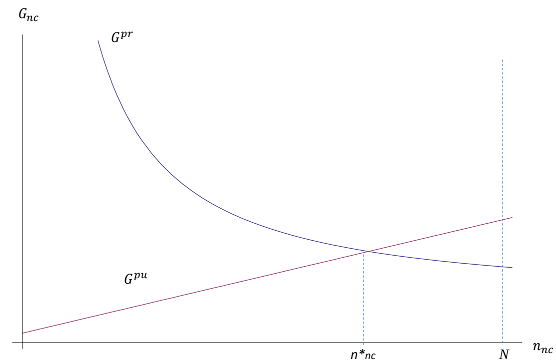


Figure 1: Equilibrium number of researchers in public lab

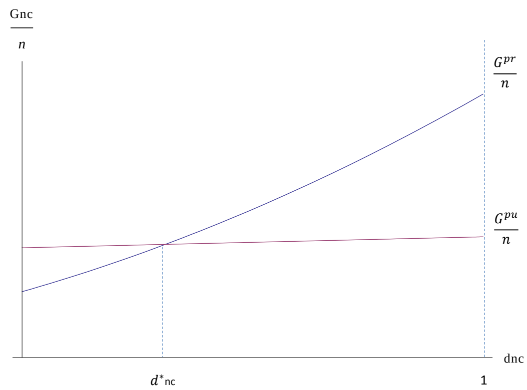


Figure 2: Equilibrium distance in a non cooperative case

From Proposition 1, depending on the level of public subsidy four types of equilibrium outcomes of the two-stage game are possible:

1. If the subsidy is too low ($s < \underline{s}$), the public payoff is always lower than the private payoff whatever the level of $n \in [0, N]$ so that N researchers are employed by the private lab.

2. If the subsidy is such that $s = \underline{s}$, there exists a unique value of $n^* = N$ such that the public payoff is equal to the private one. In this case, $d^*_{nc} = 0$ but all researchers are employed in the public lab and the private one disappears. We can deduce that the 'Research Cluster' ($d^*_{nc} = 0$) may not emerge as an equilibrium of the game (Corollary 1).

3. If $\underline{s} < s \leq \overline{s}$, an equilibrium ensuring that both private and public labs exist with $0 < d \leq 1$. Since n^* defines the number of researchers employed in the public lab, the remaining $(N - n^*)$ researchers work in the private lab. In the NC scenario, research in private lab is more profitable than research in public lab when there are few researchers in pu (see Figure 1). The low number of researchers working in pu may incite the private lab, which benefits from spillovers, to move closer to the public lab. As the number of researchers in pu increases, this creates an incentive for pr to move closer (cf lemma 1). d^*_{nc} is derived from the optimal number of researchers in each lab so that both payoffs are equal (see Figure 2)[5].

4. If the subsidy is too high ($s > \overline{s}$), the private lab chooses the maximum distance between both labs and an equilibrium exists with both labs. In this case, the spillover effect disappears but it is more efficient to benefit from the diminution of the competition cost effect by increasing the distance.

Figure 3 depicts the areas that characterize the different equilibria in the non-cooperative case for the pairwise of parameters (s,a), i.e. the public subsidy[6] and the difference in valorisation capability between labs. A high level of subsidy undoubtedly prevents the existence of a two lab equilibrium with proximity because the private lab chooses to move away to avoid the competition cost. A low level of subsidy (white area) is also harmful for a two lab equilibrium since one of the lab is not viable. A two lab equilibrium is more likely to be the outcome of the game for a low asymmetry in valorisation capability. More precisely, a quite similar valorisation capability allows a

[5]When $s = \bar{s}$, both payoffs are equal for $d^*_{nc} = 1$. In this case, we obtain $n^*_{nc} = N\frac{a}{4+a}$ which corresponds to a different solution from equilibrium 4).

[6]We do not focus on the parameter w similar in each lab to concentrate our analysis on both the level of public subsidy and the asymmetry in 'valorisation capability'. The different areas of non-cooperative equilibria are depicted in Figure 3 for $w = 20$.

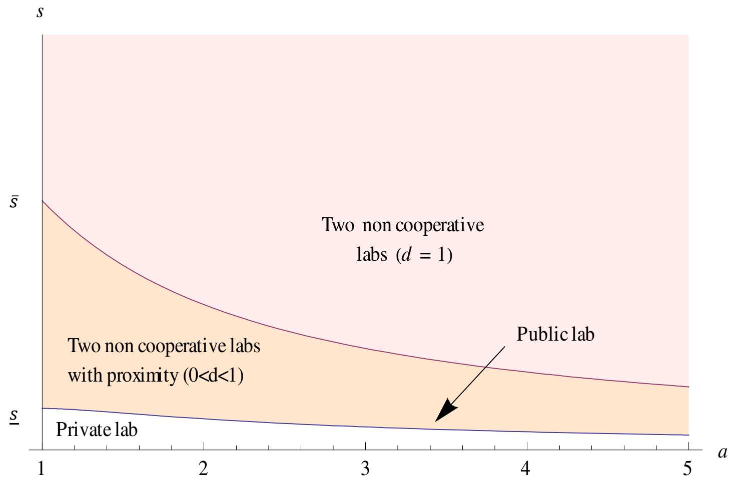


Figure 3: Non cooperative equilibria

large range of public subsidy ensuring the co-existence of two close labs. For an important asymmetry in the 'valorisation capability', the range of public subsidy ensuring the co-existence of two close labs is pretty small.

3.2 Cooperative research between public and private labs labs

In the cooperative scenario, denoted by C, we consider a situation in which labs coordinate their research efforts. Individual efforts are chosen so that they maximize the joint payoff of the labs:

$$\max_{x_i} (G^{pu} + G^{pr})$$

$$\frac{\partial G_c}{\partial x_i^{pr}} = 0 => x_{i,c}^{pr} = w\frac{(d+1)}{2} \tag{18}$$

$$\frac{\partial G_c}{\partial x_i^{pu}} = 0 => x_{i,c}^{pu} = \frac{w}{a}\frac{(d+1)}{2}(1 + a(1-d)) \tag{19}$$

We deduce the collective research effort in each lab:

$$X_c^{pr} = (N - n_c)x_c^{pr} = (N - n_c)w\frac{(d+1)}{2} \tag{20}$$

$$X_c^{pu} = n_c x_c^{pu} = n_c \frac{w}{a} \frac{(d+1)}{2} (1 + a(1-d)) \tag{21}$$

On the one hand, the expression of the individual effort in the private lab is not modified (Equations (5) and (18)). On the other hand, compared with Expression (6), it is obvious that the cooperative choice implies an additional spillover effect (Equation 19). Now, the individual effort in the public lab depends positively on the distance through the competition effect while it depends negatively on the distance through the spillover effect. By maximizing the payoff of the research labs jointly (public and private), the externalities from the public to the private lab are internalized and spillovers affect the individual effort in the public lab. The total effect depends on the value of the distance: a high distance d implies a negative effect on the public individual effort so that the spillover effect dominates the competition effect.

By replacing the optimal values of research efforts (18) and (19), we rewrite the payoff functions (2) and (4) of both labs:

$$G_c^{pu} = n \left(s + \frac{1}{4} \left(\frac{w}{a} \right)^2 (d+1) \left(1 - a^2 (1-d)^2\right) \right) \tag{22}$$

$$G_c^{pr} = w^2 (d+1) \left(\frac{(N-n)}{2} + \frac{n}{a} (1-d) (1 + a(1-d)) \right) \tag{23}$$

The private lab chooses non-cooperatively the optimal distance from the public lab, d_c, by maximising its payoff function with respect to the distance d.

$$\frac{\partial G_c^{pr}}{\partial d} = 0 \Longleftrightarrow \underbrace{\frac{1}{2}(\frac{N}{n} - 1)}_{\text{competition effect}} = \underbrace{\frac{1}{a}(2d_c + a(1+3d_c)(1-d_c))}_{\text{spillover effect}} \tag{24}$$

The analysis of the private payoff functions gives the following lemma:

Lemma 2. *For a given n, the optimal distance d_c verifies the following properties:*

- $d_c = d_{int} \in \left[0; \frac{1}{2a}\right)$ *where d_{int} is solution of* $\frac{\partial G_c^{pr}}{\partial d} = 0$ *if and only if* $n > \overline{n}$

  with $\overline{n} = N \frac{2a^2}{2a^2 + (1+a)^2}$ *and* $\frac{\partial d_{int}}{\partial n} < 0$
- $d_c = 1$ *otherwise*

Proof: in Appendix 3. □

According to the expression of individual efforts in both labs and the distance effect on theses individual efforts, we can immediately state that when the distance impacts negatively the individual public effort and also the spillover effect, the total effect through the public lab on the private payoff is positive and the private payoff is increasing with the distance. Then, the only solution is to choose the greatest distance. An interior solution may exist when the positive effect of distance on the individual effort in the public lab compensates the positive effect on the individual effort in the private lab. This depends on the number of researchers in the public lab that must be sufficient high to give a sufficient weight to the negative spillover effect in order to compensate the positive individual effort effect.

According to Definitions (11) and (12), we have the following equilibrium condition in C scenario:

$$G_c^{pu}(n_c^*) = G_c^{pr}(n_c^*)$$

By Lemma 2, both cases $d_c = d_{int}$ and $d_c = 1$ must be studied since the function G^{pr} depends on the level of n.

For the interior solution, the cooperative payoff functions, taken into account the two previous stages of the game, rewrite:

$$G_c^{pu} = n_c \left(s + \frac{1}{4} \left(\frac{w}{a} \right)^2 (d_c + 1) \left(1 - a^2 (1 - d_c)^2 \right) \right) \tag{25}$$

$$G_c^{pr} = n_c \left(w^2 \frac{(d_c + 1)^2}{2a} (1 + 2a(1 - d_c) \right) \tag{26}$$

with d_c solution of (24).

For the corner solution, the payoff functions in C are the same as in NC (16) and (17) since the spillover effect disappears.

Proposition 2. *Under C scenario, the subgame perfect Nash equilibrium is such that:*

1. *If the public subsidy s is "too low" ($s < \underset{\sim}{s}$), only the private research lab exists, $n^* = 0$.*

2. *If the public subsidy s is neither "too high" ($s < \tilde{s}$) nor "too low" ($s \geq \underset{\sim}{s}$), both research labs coexist with:*

$$n_c^* = N \frac{a}{2d_c^*(2 + 2a - 3ad_c^*) + 3a} \quad with \quad \bar{n} < n_c^* < N$$

and

$$and \quad d_c^* \in \left[0; \frac{1}{2a}\right)$$

3. *If the public subsidy s is "too high" ($s \geq \tilde{s}$), both research labs coexist with:*

$$n^*(1) = N \frac{\frac{1}{2}(w)^2}{s + \frac{1}{2}\left(\frac{w}{a}\right)^2(1+a^2)} \quad with \quad 0 < n^*(1) \leq \bar{n}$$

With

$$\tilde{s} = \left(\frac{w}{a}\right)^2 \frac{(10a-3)(1+2a)^2}{32a} \quad and \quad \underset{\sim}{s} = \left(\frac{w}{a}\right)^2 \frac{5a^2+2a-1}{4}$$

Proof: in Appendix 4. $\square$

Corollary 2. *Under C scenario, the 'pure' RC ($d_c^* = 0$ with both labs) may emerge as an equilibrium of the game when $s = \underset{\sim}{s}$. In this case, the optimal number of researchers in the public lab is constant and equals to:*

$$n_c^* = \frac{N}{3}$$

The 'pure' RC equilibrium exhibits total research efforts:

$$X_c^{pr}(d_c^* = 0) = w\frac{N}{3} \quad and \quad X_c^{pu}(d_c^* = 0) = w\frac{N}{3}\left(\frac{1+a}{2a}\right)$$

and total payoff:

$$G_c^*(d_c^* = 0) = w^2\frac{N}{3}\left(\frac{1+2a}{2a}\right)$$

Figure 4 depicts the equilibrium areas in the cooperative case. When $s = \underset{\sim}{s}$, we verify that $d_c^* = 0$; in contrast when $s = \tilde{s}$, the distance between labs is maximal ($d^* = 1$). For a level of subsidy lying between $\underset{\sim}{s}$ and $\tilde{s}$, we find that there is a continuum of cases where the private lab will choose to locate at intermediate positions. Without cooperation (or coordination of research efforts), the 'pure' RC can not exist (Corollary 1). Under C scenario, an equilibrium that consists only the public research lab may not emerge. Cooperation is necessary to guarantee the existence of the equilibrium of a 'pure' research cluster which allows the minimal proximity between labs

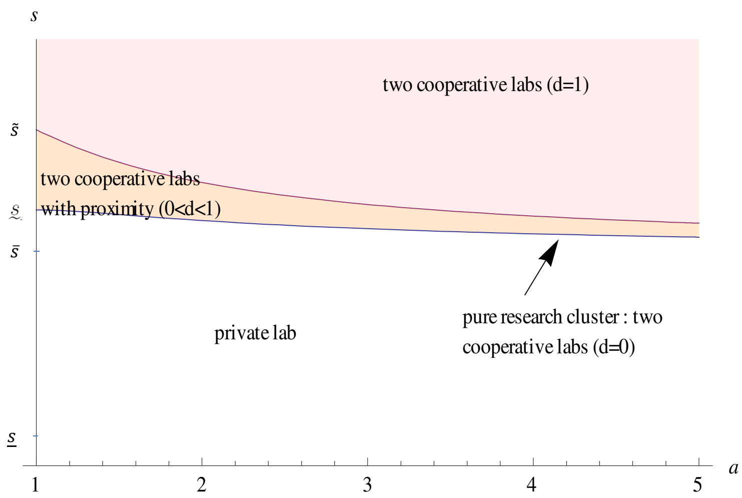


Figure 4: Cooperative equilibria

($d_c^* = 0$). The sustainability of 'pure' research cluster depends on the subsidy and organizational capability of each lab in terms of valorisation of research efforts which correspond to $\underset{\sim}{s}$ (Corollary 2). The higher the asymmetry of 'valorisation capability', the higher the public subsidy $\underset{\sim}{s}$. In this case, the total effort of the private lab and the optimal size of labs are constant: only a third of researchers will work in the public lab. The public lab efforts and the total payoff are decreasing with the asymmetry in 'valorisation capability'.

4 Comparative results

In this section, we proceed to comparisons between the cooperative and the non cooperative cases.

Let us start by comparing the impact of the public subsidy on the different variables at the equilibrium.

Corollary 3. *A rise in the public subsidy has the following effects:*

	$\frac{\partial n^*}{\partial s}$	$\frac{\partial d^*}{\partial s}$	$\frac{\partial x^*}{\partial s}$	$\frac{\partial X^{pr*}}{\partial s}$	$\frac{\partial X^{pu*}}{\partial s}$	$\frac{\partial G^*}{\partial s}$
NC (int.sol.)	<0	>0	>0	>0	$\begin{cases} <0 & if\, a<4 \\ >0 & if\, a>4 \end{cases}$	$\begin{cases} >0 & if\, s>s_G \\ \leq 0 & if\, s\leq s_G \end{cases}$
C (int.sol.)	<0	>0	>0	>0	<0	<0
Corner $(d=1)$	<0	$=0$	$=0$	>0	<0	>0

where G^* *is the payoff of each lab (identical at the equibrium) and* $s_G = \left(\frac{w}{a}\right)^2 \frac{(4-a)(4a-a^2-1)}{8}$

Proof: in Appendix 5. □

Let us start by the effects that work similarly in both the non cooperative (NC) and cooperative (C) cases, (i.e. the effect of the public subsidy on the number of researchers in each lab, the distance, the individual research efforts and the total effort in the private lab). The public subsidy has a negative effect on the equilibrium level of researchers employed in the public lab. By raising the public subsidy, the public lab's payoff becomes higher than the private lab's payoff. The labour force ensures a balance between the labs payoffs so that there are more researchers in the private lab at the expense of the public one. For the new partition of labour, both payoffs equate and both labs may coexist. As a consequence, the private lab remains distant from the public one to reestablish the balance between the distance cost competition effect and the distance spillover effect. Because there is less competition among researchers, a public subsidy encourages an increase in individual efforts (x_i) in each lab. Whichever the case (NC or C) and research organisation (public or private), the greater the distance between the labs, the greater the individual efforts. The collective effort in the private lab increases with the rise in the number of researchers and their individual efforts. Now, let us comment on the effect of the public subsidy on the total effort in the public sector. In the non cooperative case, when the asymmetry in 'valorisation capability' between the public and the private lab is low (i.e. $a < 4$), the decline in the size of public lab has a negative impact on its total effort that cannot be offset by any increase of individual efforts. When the difference in 'valorisation capability' is sufficiently high, then this decline is more than offset by the individual effort reaction. In the cooperative equilibrium, the result is clearcut: any increase in the public subvention decreases the total effort in public sector; the decline in the size of public lab always dominates the increase of individual efforts.

Finally, the payoffs decrease with the public subsidy at the cooperative equilibrium while it can either decrease or increase at the non cooperative equilibrium depending on the level of subsidy. In the non cooperative case, the positive effect on distance must be supported by a sufficient high public subsidy to allow for a rise in the payoff. At the cooperative equilibrium, the number of researchers effect (< 0) clearly dominates the distance effect (> 0). In the extreme case of the corner solution, the maximum distance ($d = 1$) allows the individual efforts of researchers to significantly intensify (no competition effect): a rise in the public subsidy increases the equilibria payoffs.

Let us now compare the equilibrium payoffs in the NC and C scenarii. The equilibrium areas in the cooperative and non-cooperative cases are depicted in Figure (5). A high level of subsidy prevents the existence of an equilibrium with proximity irrespective of whether the both labs cooperate or not (Area 6). By contrast, a low level of subsidy (Area 1) only enables the private lab to maintain the research activity. For intermediate levels of subsidy, the cooperative and non-cooperative cases do not achieve the same equilibrium results. Therefore, we have:

- **Area 2**: public lab alone (NC) *vs* private lab alone (C),
- **Area 3**: two close non-cooperative labs (NC) *vs* private lab alone (C),
- **Area 4**: two non-cooperative labs with $d = 1$ (NC) *vs* private lab alone (C),
- **Area 5**: two close cooperative labs (C) *vs* two non-cooperative labs with $d = 1$ (NC).

It is not clear, *a priori*, which equilibrium will imply a higher payoff in each Area (2-5). By comparing the labs payoffs in each area, we obtain the following proposition:

Proposition 3. *There exist threshold levels of subsidy s such that:*

- ***Area 2*** *($s = \underline{s}$): when $a \leq 2$, the non cooperative equilibrium payoff (public lab) dominates the cooperation equilibrium payoff (private lab) ($G_A^{pu} \geq G_A^{pr}$); for $a > 2$, the opposite result applies ($G_A^{pu} < G_A^{pr}$),*
- ***Area 3*** *($\underline{s} < s < \overline{s}$): if $\underline{s} < s_1 < s < \overline{s}$ the non cooperative equilibrium payoff (two close labs, $d < 1$) dominates the cooperation equilibrium payoff (private lab alone): $G_{nc}^{*} > G_A^{pr}$; if $\underline{s} < s < s_1 < \overline{s}$ the opposite results applies.*

- ***Area 4*** *($\bar{s} \leq s < \underset{\sim}{s}$): the non cooperative equilibrium payoff (Corner, $d = 1$) always dominates the cooperation equilibrium payoff (private lab alone): $G^*_{nc}(d = 1) > G^{pr}_A$,*[7]

- ***Area 5*** *($\underset{\sim}{s} \leq s < \tilde{s}$): if $\underset{\sim}{s} \leq s < s_2 < \tilde{s}$, the cooperative equilibrium payoff (two close labs, $d < 1$) dominates the non-cooperation equilibrium payoff (Corner, $d = 1$): $G^*_c > G^*_{nc}(d = 1)$; if $\underset{\sim}{s} < s_2 \leq s < \tilde{s}$ the opposite result applies.*

where $s_1 = \left(\frac{w}{a}\right)^2 \frac{a^2-2a-1+(2a-1)\sqrt{a-3}}{8}$ and s_2 is given by simulation[8].

Proof: See Appendix 5. □

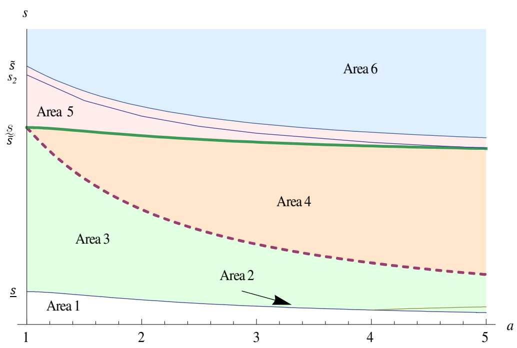
According to Proposition 3, there are only two areas (Area 2 and Area 5) in which the payoff of the cooperative equilibrium may dominate the payoff of the non cooperative equilibrium. Note that in Area 5, for $s_2 < s < \tilde{s}$ the non-cooperative equilibrium payoff dominates the cooperative one. This quite surprising result is partly due to the nature of the equilibria that are strongly determined by the couple of parameters (a, s). As Figure (5) depicts, there is no couple (a, s) that allows for an equilibrium with two labs and proximity under both the non-cooperative and the cooperative strategy as the curve $\underset{\sim}{s}$ (in bold) is always above the curve $\bar{s}$ (in dash).

As depicted by Figure (6), whichever research behavior adopted (cooperative or not), a high asymmetry in the 'valorisation capability' between public and private lab does not favor spatial proximity. Compared to the non-cooperative one, it is clear that the cooperative area is very reduced especially when the parameter a is high. Finally, for all asymmetries in valorisation capability, the level of public subsidy allowing for the existence of clusters is always higher than in the non cooperative case, $\bar{s} \leq \underset{\sim}{s}$. For the same valorisation capability ($a = 1$), we verify that $\bar{s} = \underset{\sim}{s}$. In this case, the 'pure' cluster with $d^*_c = 0$ emerges as an equilibrium of the game.

In the following Corollary (4) we concentrate our analysis on the 'pure' research cluster performances:

[7] This result stands for any $a < 12$. We have decided to rule out the unlikely cases where $a > 2$

[8] Since we do not have any analytical values of the optimal distance and number of researchers in each lab in the cooperative case, we are not able to produce analytical results for the comparison of the payoff. We proceed to simulations to compare them.



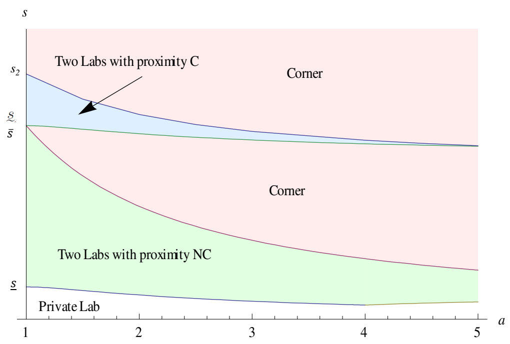


Figure 5: Cooperative and non-cooperative equilibria.

Figure 6: Payoff equilibria dominance.

Corollary 4. *For $s = \underline{s}$, the cooperative payoff resulting from the 'pure' RC equilibrium ($d_c^* = 0$ with both labs) dominates the non-cooperative equilibrium payoff (corner solution, i.e. $d = 1$) and we verify:*

- $n_c^*(d_c = 0) > n(1)$
- $x_c^{pu*}(d_c = 0) > x^{pu}(d = 1)$ *and* $x_c^{pr*}(d_c = 0) = x^{pr}(d = 1)$
- *a)* $X_c^{pu*}(d_c = 0) > X^{pu}(d = 1)$

  b) $X_c^{pr*}(d_c = 0) < X^{pr}(d = 1)$

  c) $X_c{}^*(d_c = 0) < X(d = 1)$

A 'pure' research cluster equilibrium implies a bigger public lab than a non cooperative equilibrium with $d = 1$. In addition, we find that a 'pure' RC does not affect the individual research effort in the private lab but encourages the researchers' efforts in the public lab. Consequently, the total research effort in public lab at the 'pure' RC equilibrium is greater than the total public effort at the non-cooperative equilibrium. However, compared to non-cooperative equilibrium, the improvement on research efforts in public lab is not enough to counterbalance the decrease of the total research effort in the private lab (since the number of researchers in the private lab is lower than a non cooperative equilibrium). Although the total output gains are better in the 'pure' RC, this performance may be moderated by the results in terms of research efforts. The research cluster facilitates spatial proximity and can yield a higher output gain that can be appropriated by each lab than in non-cooperation. But, we find that a 'pure' RC cannot perform better in terms of total research efforts, even if it allows to encourage the public research effort.

5 Conclusion

Only a few years ago, the conventional wisdom predicted that research clusters would lead to foster public-private research linkages that may help firms to internalize the knowledge spillovers from the academic research sector, resulting in greater research efforts. The obsession of policy-makers in developed countries to 'create the next Silicon Valley' reveals the increased importance of spatial proximity in innovative activity. However, public authorities cannot force firms and other research actors to cooperate and to choose the closest location. The implementation of cluster policies is complex and it is actually difficult to know how to target such cluster policies (Duranton, 2007; Falck et al., 2010; Fontagné et al., 2013; Duranton et al., 2010). Thus, the first interesting question that arises is under which conditions the private research sector opts for the spatial proximity to universities. If the ability to receive knowledge spillovers is influenced by the distance from the knowledge source (as pointed out by several empirical studies), the spatial concentration should always be observed. However, the propensity for innovative activity to cluster geographically varies across industries, countries, regions, etc. Another question concerns the performance of the research cluster: does the cooperation with proximity (RC) lead to better performance than the non-cooperation with distant location?

In this paper, we developed a two-stage game for a better understanding of the endogenous formation of a research cluster. The model highlights the role of public subsidies and valorization capabilities in the emergence of research clusters. By determining the equilibria of the non cooperative and cooperative cases, we are able to depict the range of public subsidies allowing for the co-existence of two close labs for a given valorization capability. We show that a low level of the public subsidy is harmful for the existence of the public research while a high level of the public subsidy implies the highest distance between labs; the higher the asymmetries in valorisation capabilities between the public and the private labs, the more difficult it is to ensure the research cluster formation.

We may retain two major lessons from our results in order to provide guidelines for research policies: (i) increase the level of public subsidy is not necessarily efficient to sustain RC. This reduces the positive spillovers from the public research sector to the private research sector and leads the private research lab to locate further away from public research lab; (ii) if the objectives of a policy-maker is to ensure greater production and better dissemination of new knowledge, cluster policies are not the panacea: a RC provides a stimulus to research efforts in the public lab at the expense of the private lab. At the aggregate level, the performance is lower than the non cooperative one in terms of research efforts.

In addition, we should note that these results have been obtained in the context of interaction between only two labs without taking into account the market of knowledge (valorization capability in research is given by exogenous parameters). Our analysis can be seen as a first step in the understanding of the formation of research clusters. We still need new insights from theory to better assess the existence conditions and the performance of research clusters.

Appendix 1: Proof of Lemma 1

Equation 9 is equivalent to $d_{nc} = \frac{a(N-n)}{4n}$ which is verified if and only if

$$0 \leq \frac{a\left(N-n\right)}{4n} \leq 1$$

The left hand side is always checked if and only if $n \leq N$, while the right hand side holds if and only if $n \geq N\frac{a}{4+a}$.
If $n < N\frac{a}{4+a}$, the optimal solution is to choose $d = 1$ which corresponds to the corner solution.

Appendix 2: Proof of Proposition 1

According to (13), an equilibrium with both labs and an interior solution for d exists if and only if

$$D_{nc} = G_{nc}^{pu} - G_{nc}^{pr} = 0 \Longleftrightarrow n\ \left(s + \frac{1}{4}\left(\frac{w}{a}\right)^2\left(2+d_{nc}\right)\right) = \frac{n}{2a}\left(w\right)^2\left(1+d_{nc}\right)^2$$

We define

$$F\left(d_{nc}\right) = s + \frac{1}{2}\left(\frac{w^{pr}}{a}\right)^2\left(1+d_{nc}\right)\left(\frac{1}{2} - a\left(1+d_{nc}\right)\right)$$

and d_{nc}^{*} is solution of $F\left(d_{nc}^{*}\right) = 0$. Let us study function F:

$$F'\left(d_{nc}\right) = \frac{1}{2}\left(\frac{w^{pr}}{a}\right)^2\left(\frac{1}{2} - 2a\left(1+d_{nc}\right)\right) < 0\ \forall a > 1$$

$$F"\left(d_{nc}\right)=-a\left(\frac{w}{a}\right)^{2}<0$$

$$F\left(0\right)=s+\frac{1}{2}\left(\frac{w^{pr}}{a}\right)^{2}\left(\frac{1}{2}-a\right)>0\Leftrightarrow s>\left(\frac{w^{pr}}{a}\right)^{2}\left(\frac{2a-1}{4}\right)=\underline{s} \quad (27)$$

$$F\left(1\right)=s+\left(\frac{w^{pr}}{a}\right)^{2}\left(\frac{1}{2}-2a\right)<0\Longleftrightarrow s<\left(\frac{w^{pr}}{a}\right)^{2}\left(\frac{4a-1}{2}\right)=\overline{s} \quad (28)$$

F is always decreasing. There exists a $d_{nc}^{*}\in(0,1]$ such that $F\left(d_{nc}^{*}\right)=0$ if and only if $F\left(1\right)\leq 0$ that holds when $s\leq\overline{s}$ and $F(0)>0$ when $s>\underline{s}$. We can deduce:

- When $s<\underline{s}$, Equation (13) is satisfied for a $n_{nc}>N$ which induces that G_{nc}^{pr} is always superior to G_{nc}^{pu} for any $0\leq n\leq N$. Consequently, only the private lab remains and $n_{nc}^{*}=0$.
- When $s=\underline{s}$, from Equations (9) and (13) we have $n_{nc}^{*}=N$. Thus only the public lab exists.
- When $\underline{s}<s\leq\bar{s}$, the analysis of Function F shows that there exists a unique subgame perfect Nash equilibrium, $0<n_{nc}^{*}<N$, which implies that $d_{nc}^{*}\in(0,1]$:

  n_{nc}^{*} is obtained by equation :

  $$n_{nc}^{*}=N\frac{a}{4d_{nc}^{*}(n_{c}^{*})+a} \quad \text{with} \quad N\frac{a}{4+a}\leq n_{nc}^{*}<N \quad \text{for } d_{nc}^{*}\in(0,1]$$

- When $s>\overline{s}$, n_{nc} resulting from (13) is such as $d_{nc}\geq 1$. As $n_{nc}<N\frac{a}{4+a}$, the optimal solution is to choose $d^{*}=1$ which corresponds to the corner solution. Thus,

  $$n^{*}\left(1\right)=N\frac{\frac{1}{2}\left(w\right)^{2}}{s+\frac{1}{2}\left(\frac{w}{a}\right)^{2}\left(1+a^{2}\right)} \quad \text{with} \quad 0<n^{*}\left(1\right)<N\frac{a}{4+a} \quad (29)$$

Appendix 3: Proof of Lemma 2

Let us study the form of the private payoff function according to d_c

$$\frac{\partial^{2}\left(G_{c}^{pr}\right)}{\partial d^{2}}=-\frac{2n}{a}(1+a-3ad)$$

which implies that the private payoff is concave on $d\in\left(0,\frac{1+a}{3a}\right]$ and convex on $d\in\left[\frac{1+a}{3a},1\right]$.

Let us show that $0 < d_{int} < \frac{1+a}{3a}$.

If an interior solution exists, the level of d is given by the relation

$$\frac{\partial G_c^{pr}}{\partial d} = 0$$

$$\Longleftrightarrow \frac{1}{2}(N-n) = \frac{n}{a}(2d_{int}(1+a(1-d_{int})) + a(1+d_{int})(1-d_{int})) \quad (30)$$

Thus, we deduce the relation between n and d for a given n:

$$n = N\frac{a}{2d_{int}(2+2a-3ad_{int})+3a} \quad (31)$$

with $n \in (0, N)$ for any $d \in [0,1]$ [9].

Now let us compare $G_c^{pr}(1)$ and $G_c^{pr}(d_{int})$.

$$G_c^{pr}(d_{int}) - G_c^{pr}(1) = (w^{pr})^2(1-d_{int})^2\frac{n}{a}(1-2ad_{int}) > 0 \Longleftrightarrow d_{int} < \frac{1}{2a}$$

Let check that $d_{int} < \frac{1}{2a}$.

$$\left.\frac{\partial G_c^{pr}}{\partial d}\right|_{d_{int}=\frac{1}{2a}} = \frac{1}{2}\left(N - n\left(\frac{(2a+1)^2}{2a^2}+1\right)\right)$$

We can state that $d_{int} < \frac{1}{2a}$ if $\left.\frac{\partial G_c^{pr}}{\partial d}\right|_{d_{int}=\frac{1}{2a}} < 0$, since we are located on the concave part of the curve described by $\frac{\partial G_c^{pr}}{\partial d}$

$$\left.\frac{\partial G_c^{pr}}{\partial d}\right|_{d_{int}=\frac{1}{2a}} < 0 \Leftrightarrow n > N\frac{2a^2}{2a^2+(1+2a)^2} = \overline{n} \quad (32)$$

From (31), we obtain

$$\frac{\partial d_c}{\partial n_c} = -\left(\frac{(1+2a-3ad)aN}{[d_c(4+8a-6ad_c)-(a+2)]^2}\right)^{-1} < 0 \Longleftrightarrow d < \frac{1+2a}{3a}$$

which is verified for an interior solution.

Consequently, $d_{int} \in \left[0; \frac{1}{2a}\right)$ where d_{int} is solution of $\frac{\partial G_c^{pr}}{\partial d} = 0$ if and only if $n > \overline{n}$. When condition (32) is not satisfied, the corner solution holds: $d_c = 1$ since

$$G_c^{pr}(d_{int}) \leq G_c^{pr}(1)$$

[9] We have $n < N \Longleftrightarrow \frac{1+a-\sqrt{1+2a+4a^2}}{3a} < 0 < d_{int} \leq 1 < \frac{1+a+\sqrt{1+2a+4a^2}}{3a}$

Appendix 4: Proof of Proposition 2

According to (25) and (35) we can state that and interior solution exists if

$$\frac{G_c^{pu}}{n} - \frac{G_c^{pr}}{n} = 0$$

We have to analyze the following function:

$$H(d) = \frac{G_c^{pu}}{n} - \frac{G_c^{pr}}{n}$$

$$= \left(s + \frac{1}{4}\left(\frac{w}{a}\right)^2 (d+1)\left(1 - a^2(1-d)^2\right)\right) - \left(w^2\left(\frac{(d+1)^2}{2a}\right)(1 + 2a(1-d))\right)$$

We obtain:

$$H'(d) = \left(\frac{w^{pr}}{a}\right)^2 \frac{1}{4}\left(1 - 4a(1+d) + a^2\left(9d^2 + 10d - 3\right)\right)$$

$$H'(0) = \left(\frac{w^{pr}}{a}\right)^2 \frac{1}{4}\left(1 - 4a - 3a^2\right) < 0$$

$$\begin{aligned} H'\left(\frac{1}{2a}\right) &= \left(\frac{w^{pr}}{a}\right)^2 \frac{1}{4}\left(1 - 4a\left(1 + \frac{1}{2a}\right) + a^2\left(\frac{9}{4a^2} + \frac{5}{a} - 3\right)\right) \\ &= \left(\frac{w^{pr}}{a}\right)^2 \frac{1}{16}\left(5 + 4a - 12a^2\right) < 0 \end{aligned}$$

$$H''(d) = \left(\frac{w}{a}\right)^2 \frac{1}{4}\left(18a^2 d + 10a^2 - 4a\right) > 0$$

Then $H(d_{int})$ is decreasing for any $d \in \left[0; \frac{1}{2a}\right)$.
We now determine the value of $H(d_{int})$ for the boundary values of d:

- If $H\left(\frac{1}{2a}\right) \geq 0$, we have $H(d) \geq 0$ for any $d_{int} \in [0,1)$ and we have $G_c^{pu} \geq G_c^{pr}$, for $0 < n < N$. In this case, any interior solution exists. Then, the corner solution $d_c = 1$ holds when:

$$H\left(\frac{1}{2a}\right) \geq 0$$

$$\iff s > \widetilde{s} = \frac{(10a-3)(1+2a)^2}{32a}\left(\frac{w}{a}\right)^2 \tag{33}$$

  The optimal level of n is given by $G_c^{pu}(d_c = 1) = G_c^{pr}(d_c = 1)$ which implies $n^*(1)$.

- If $H(0) < 0$, we have $H(d) < 0$ and $G_c^{pr} > G_c^{pu}$ for $0 < n < N$. Then $n^* = 0$ and only the private lab exists. We have

$$H(0) = s - \frac{5a^2 + 2a - 1}{4} \left(\frac{w}{a}\right)^2 < 0$$
$$\Longleftrightarrow s < \frac{5a^2 + 2a - 1}{4} \left(\frac{w}{a}\right)^2 = \underset{\sim}{s} \tag{34}$$

- If $H(0) = 0$, we have

$$G_c^{pr}(d = 0) = G_c^{pu}(d = 0)$$
$$\Longleftrightarrow s = \underset{\sim}{s}$$

We obtain an interior solution with $n^* = \frac{N}{3}$.

- From the three previous points, we can state that if $\underset{\sim}{s} \leq s < \tilde{s}$, there exists a cooperative interior solution value with $\bar{n} < n_c^* < N$ and $d_c^* \in \left[0; \frac{1}{2a}\right)$

Appendix 5: Proof of Corollary 3

i) The non-cooperative scenario:
According to (13), an equilibrium in NC with both research labs and an interior solution for d exists if and only if

$$s + \frac{1}{4}\left(\frac{w}{a}\right)^2 (1 + d_{nc}^*) = \frac{w^2}{2a}(1 + d_{nc}^*)^2$$

The total derivative of the above expression with respect to s gives:

$$1 + \frac{1}{4}\left(\frac{w}{a}\right)^2 \frac{\partial d_{nc}^*}{\partial s} = \left(\frac{w^2}{a}\right)(1 + d_{nc}^*)\frac{\partial d_{nc}^*}{\partial s}$$
$$\frac{\partial d_{nc}^*}{\partial s} = \left(\frac{a}{w}\right)^2 \left(\frac{4}{4a\,(1 + d_{nc}^*) - 1}\right) > 0$$

from $n_{nc}^* = N\frac{a}{4d_{nc}^* + a}$, we have

$$\frac{\partial n_{nc}^*}{\partial s} = -n_{nc}^* \frac{4}{4d_{nc}^* + a}\frac{\partial d_{nc}^*}{\partial s} < 0$$

Obviously, from Equations (5), (6) and (7), we have

$$\frac{\partial x_{nc}^{pr*}}{\partial s} > 0, \quad \frac{\partial x_{nc}^{pu*}}{\partial s} > 0 \quad \text{and} \quad \frac{\partial X_{nc}^{pr*}}{\partial s} > 0$$

$$\begin{aligned}
\frac{\partial X_{nc}^{pu*}}{\partial s} &= \frac{1}{2}\frac{w}{a}\left(\frac{\partial n_{nc}^*}{\partial s}\left(1+d_{nc}^*\right)+n_{nc}^*\frac{\partial d_{nc}^*}{\partial s}\right)\\
&= \frac{1}{2}\frac{w}{a}\left(-n_{nc}^*\frac{4}{4d_{nc}^*+a}\frac{\partial d_{nc}^*}{\partial s}\left(1+d_{nc}^*\right)+n_{nc}^*\frac{\partial d_{nc}^*}{\partial s}\right)\\
&= \frac{1}{2}\frac{w}{a}n_{nc}^*\frac{\partial d_{nc}^*}{\partial s}\left(\frac{a-4}{4d_{nc}^*+a}\right)>0 \Longleftrightarrow a>4
\end{aligned}$$

and we deduce the effect on $G_{nc}^* = \frac{1}{2}\frac{w^2}{a}n_{nc}^*\left(1+d_{nc}^*\right)^2$

$$\begin{aligned}
\frac{\partial G_{nc}^*}{\partial s} &= \frac{1}{2}\frac{w^2}{a}\left(\frac{\partial n_{nc}^*}{\partial s}\left(1+d_{nc}^*\right)^2+2n_{nc}^*\frac{\partial d_{nc}^*}{\partial s}\left(1+d_{nc}^*\right)\right)\\
&= \frac{1}{2}\frac{w^2}{a}\frac{\partial d_{nc}^*}{\partial s}\frac{\left(1+d_{nc}^*\right)n_{nc}^*}{\left(4d_{nc}^*+a\right)}2\left(2d_{nc}^*+a-2\right)>0 \Longleftrightarrow d_{nc}^*>\frac{2-a}{2}
\end{aligned}$$

Replacing d_{nc}^* by its expression given in Proposition 1, we obtain:

$$\frac{\partial G_{nc}^*}{\partial s}>0 \Longleftrightarrow s>s_G=\left(\frac{w}{a}\right)^2\frac{(4-a)\left(4a-a^2-1\right)}{8}$$

ii) The cooperative scenario:
From Proposition 2,

$$G_c^{pu}(d_c^*)=G_c^{pr}(d_c^*)$$

$$\Leftrightarrow s+\frac{1}{4}\left(\frac{w}{a}\right)^2\left(d_c^*+1\right)\left(1-a^2\left(1-d_c^*\right)^2\right)=\frac{1}{2}\left(\frac{w^2}{a}\right)\left(d_c^*+1\right)^2\left(1+2a\left(1-d_c^*\right)\right)$$

The total derivative of the above expression with respect to s gives:

$$ds=dd_c^*\left(-\frac{\partial\left(\frac{G_c^{pu}}{n_c^*}-\frac{G_c^{pr}}{n_c^*}\right)}{\partial d_c^*}\right)$$

We obtain from the analysis of function H in the Appendix 4 that

$$\frac{\partial d_c^*}{\partial s}=-\frac{1}{\frac{\partial H}{\partial d_c^*}}>0$$

from (31), we calculate:

$$\frac{\partial n_{nc}^*}{\partial s}=-n_{nc}^*\frac{\partial d_c^*}{\partial s}\frac{4\left(1+a-3ad_c^*\right)}{2d_c^*(2+2a-3ad_c^*)+3a}<0 \text{ since } d_c^*\in\left[0,\frac{1}{2a}\right)$$

Obviously, from Equations (18), (19) and (20), we have

$$\frac{\partial x_c^{pr*}}{\partial s} > 0, \quad \frac{\partial x_c^{pu*}}{\partial s} > 0 \quad \text{and} \quad \frac{\partial X_c^{pr*}}{\partial s} > 0$$

$$\begin{aligned}
\frac{\partial X_c^{pu*}}{\partial s} &= \frac{1}{2}\frac{w}{a}\left(\frac{\partial n_c^*}{\partial s}\left(d_c^*+1\right)\left(1+a(1-d_c^*)\right)+n_{nc}^*\frac{\partial d_c^*}{\partial s}\left(1-2ad_c^*\right)\right) \\
&= \frac{1}{2}\frac{w}{a}n_{nc}^*\frac{\partial d_c^*}{\partial s}\left(\frac{-4+7ad_c^*-5a\left(1-d_c^*\right)-2ad_c^{*2}\left(a-1\right)-2a^2\left(1-d_c^*\right)\left(2-d_c^*\right)}{2d_c^*(2+2a-3ad_c^*)+3a}\right) < 0
\end{aligned}$$

since $d_c^* \; < \; \dfrac{1}{2a}$

and we deduce the effect on $G_c^* = n_c^*\left(\frac{w^2}{a}\right)\left(d_c^*+1\right)^2\left(1+2a\left(1-d_c^*\right)\right)$

$$\begin{aligned}
\frac{\partial G_c^*}{\partial s} &= \frac{w^2}{a}\left(d_c^*+1\right)\left(\frac{\partial n_c^*}{\partial s}\left(d_c^*+1\right)\left(1+2a\left(1-d_c^*\right)\right)+n_c^*\frac{\partial d_c^*}{\partial s}\left(2\left(1+a-3ad_c^*\right)\right)\right) \\
&= \frac{2w^2}{a}\left(d_c^*+1\right)n_{nc}^*\frac{\partial d_c^*}{\partial s}\frac{\left(1+a-3ad_c^*\right)\left(-a-2\left(1-d_c^*\right)-4ad_c^*-2ad_c^{*2}\right)}{2d_{int}(2+2a-3ad_c^*)+3a} < 0
\end{aligned}$$

(iii) The Corner solution:
From Equation (29), we easily obtain that

$$\frac{\partial n^*\left(1\right)}{\partial s} < 0$$

Obviously, from Equations (7) and (8), we deduce

$$\frac{\partial X^{pr*}(d=1)}{\partial s} > 0, \quad \frac{\partial X^{pu*}(d=1)}{\partial s} < 0$$

From Equation (17), we have

$$\frac{\partial G^*\left(d=1\right)}{\partial s} > 0$$

Appendix 6: Proof of Proposition 3

Let us compare the payoff in the different Areas. We denote by G_c^* and G_{nc}^* the equilibrium payoff resulting from the cooperative and non-cooperative strategies.

- **Area 2** with $s = \underline{s}$ (given by equation (27)).
  From equations (1), (3), (6), and (18), we compare:

$$G_{nc}^{*} = G_{A}^{pu} = N\frac{1}{2}\left(\frac{w}{a}\right)^{2}\left(\frac{2a+1}{2}\right)$$

with

$$G_{c}^{*} = G_{A}^{pr} = N\frac{w}{4}^{2}$$

We obtain

$$G_{c}^{*} > G_{nc}^{*} \Longleftrightarrow 1 > \left(\frac{2a+1}{2a^{2}}\right) \Longleftrightarrow a > 2$$

- **Area 3** with $\underline{s} \leq s < \overline{s}$ (given by equations (27) and (28)).
  From equations (1), (14), (15) and (18), we compare:

$$G_{nc}^{*} = G_{nc}^{pu*} + G_{nc}^{pr*} = n\frac{w^{2}}{a}(1+d)^{2}$$

with

$$G_{c}^{*} = G_{A}^{pr} = N\frac{w}{4}^{2}$$

We obtain

$$G_{nc}^{*} \geqslant G_{c}^{*} \Longleftrightarrow \frac{n}{a}w^{2}(1+d)^{2} \geqslant N\frac{w}{4}^{2}$$

$$\Longleftrightarrow \frac{n}{a}(1+d)^{2} \geqslant \frac{N}{4}$$

$$\Longleftrightarrow 2\left(1+d_{nc}^{*}\right)^{2} \geqslant 4d_{nc}^{*} + a$$

$$\Longleftrightarrow 4d\,(d+1) \geqslant a-4$$

wich is always true for $a \leqslant 4$.
If $a > 4$ we have $G_{nc}^{*} \geqslant G_{c}^{*} \Longleftrightarrow d_{nc}^{*} \geqslant \frac{-1+\sqrt{a-3}}{2}$

replacing the value of d_{nc}^{*}, we obtain

$$\frac{1}{4a}\left[1-4a+\sqrt{1+32sa\left(\frac{a}{w^{pr}}\right)^{2}}\right] > \frac{-1+\sqrt{a-2}}{2}$$

$$\Longleftrightarrow \sqrt{1+32sa\left(\frac{a}{w^{pr}}\right)^{2}} > 2a\sqrt{a-3}+2a-1$$

$$\Longleftrightarrow 1 + 32sa\left(\frac{a}{w^{pr}}\right)^2 > \left(2a\sqrt{a-3} + 2a - 1\right)^2$$

$$\Longleftrightarrow s > \frac{4a^2(a-2) + 4a(2a-1)\sqrt{a-3} - 4a}{32a\left(\frac{a}{w^{pr}}\right)^2}$$

$$\Longleftrightarrow s > \left(\frac{w}{a}\right)^2 \frac{a^2 - 2a - 1 + (2a-1)\sqrt{a-3}}{8} = s_1$$

- **Area 4** with $\overline{s} < s < \underset{\sim}{s}$ (given by equations (28) and (34)).
  From equations (1), (6), (16), (17), (18) and (29), we compare:

$$G_c^* = G_A^{pr} = N\frac{w^2}{4}$$

with

$$G_{nc}^* = G_{nc}^{pu}(1) + G_{nc}^{pr}(1) = w^2(N - n(1))$$

We obtain

$$G_{nc}^* \quad > \quad G_c^* \Longleftrightarrow \frac{N}{4} < (N - n(1))$$

$$\Longleftrightarrow \frac{1}{4} < \left(1 - \frac{\frac{1}{2}w^2}{s + \frac{1}{2}\frac{w^2}{a^2}(1+a^2)}\right)$$

$$\Longleftrightarrow \frac{\frac{1}{2}w^2}{s + \frac{1}{2}\frac{w^2}{a^2}(1+a^2)} < \frac{3}{4}$$

$$\Longleftrightarrow \frac{w^2}{a^2}\left(\frac{a^2-3}{6}\right) < s$$

We verify that $\overline{s} > \frac{w^2}{a^2}\left(\frac{a^2-3}{6}\right) \Longleftrightarrow a > 12$. We consider that any analysis with $a > 12$ can be ruled out.

- **Area 5** with $\underset{\sim}{s} < s < \widetilde{s}$ (given by equations (34)and (33)).
  From equations (17),(25), (29) and (31), we compare

$$G_{nc}^* = 2n(1)\left[s + \frac{1}{2}\left(\frac{w^{pr}}{a}\right)^2\right]$$

with

$$G_c^* = G_c^{pu*} + G_c^{pr*}$$

where

$$G_c^{pu*} = n_c^* \left(s + \frac{1}{4} \left(\frac{w^{pr}}{a} \right)^2 (d^* + 1) \left(1 - a^2 (1 - d^*)^2\right) \right)$$

and

$$G_c^{pr*} = n_c \left(w^2 \frac{(d_c + 1)^2}{2a} (1 + 2a(1 - d_c) \right)$$

Since we do not have any analytical values of the optimal distance and number of researchers in each lab in the cooperative case, we are unable to produce analytical results for the comparison of the payoff. We proceed to simulations to compare them. Simulations are undertaken for $w = 20$. For any $s > s_2$, we have $G_{nc}^* > G_c^*$. The following table gives the value of s_2.

a	1	1.5	2	2.5	3	3.5	4	4.5	5
$\underset{\sim}{s}$	600	588.889	575	564	555.556	548.98	543.75	539.506	536
s_2	760.293	682.05	634.554	603.741	582.325	573.032	554.652	545.219	537.602
$\widetilde{s}$	787.5	711.111	664.063	633.6	612.5	597.085	585.352	576.132	568.7

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